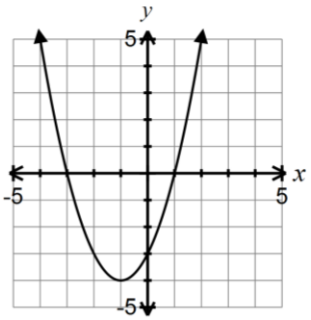
# Inequalities Involving Quadratic Functions

**Examples:** Solve each inequality using the graph of $f(x) = x^2 + 2x - 3$.

Notice that each of these inequalities involves the value of $x^2 + 2x - 3$, which is represented by the *y*-coordinate of the graph. In each case, we are trying to figure out what *x*-values (*x*-coordinates) make the inequality true. When trying to find where $x^2 + 2x - 3 > 0$, we are trying to figure out what *x*-coordinates have a *y*-coordinate that is bigger than zero—in other words, *where is the graph above the x-axis?*

$f(x) = x^2 + 2x - 3$




a) $x^2 + 2x - 3 > 0$

b) $x^2 + 2x - 3 \geq 0$

c) $x^2 + 2x - 3 < 0$

d) $x^2 + 2x - 3 \leq 0$

**Solving a Quadratic Inequality in One Variable Graphically:**

1. Write the inequality in standard form. Replace the inequality sign with an equal sign and solve the equation $ax^2 + bx + c = 0$ by factoring, completing the square, or using the quadratic formula. This gives you the *x*-intercepts of the graph of $y = ax^2 + bx + c$.
2. Graph the related function $y = ax^2 + bx + c$. The graph does not have to be very detailed. A rough sketch of a parabola opening in the correct direction with the correct *x*-intercepts is all you need.
3. The solutions of $ax^2 + bx + c > 0$ are the *x*-values for which the graph is **above** the *x*-axis.
   The solutions of $ax^2 + bx + c \geq 0$ are the *x*-values for which the graph is **on or above** the *x*-axis.
   The solutions of $ax^2 + bx + c < 0$ are the *x*-values for which the graph is **below** the *x*-axis.
   The solutions of $ax^2 + bx + c \leq 0$ are the *x*-values for which the graph is **on or below** the *x*-axis.
4. If the inequality involves $\leq$ or $\geq$, the *x*-intercepts **are included** in the solution set (use brackets).
   If the inequality involves < or >, the *x*-intercepts **are not included** in the solution set (use parentheses).

**Example:** Solve each inequality and graph the solution set on a number line.

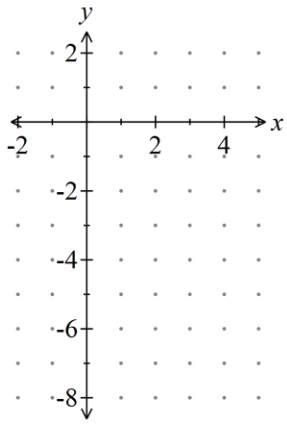
a) $x^2 - 10x + 16 \leq 0$

b) $x^2 + 2x > 0$

c) $-x^2 - 7x \geq 10$

d) $3x^2+10x-8<0$ e) $x^2-4x+5\le 0$ f) $x^2+6x+9>0$

**Example:** Solve the inequality $2x^2<5x+3$ and graph the solution set.
Method 1: Rearrange the inequality so that 0 is on the right side.




Method 2: Find the intersections of the two functions and determine where $f(x)=2x^2$ is less than $f(x)=5x+3.$